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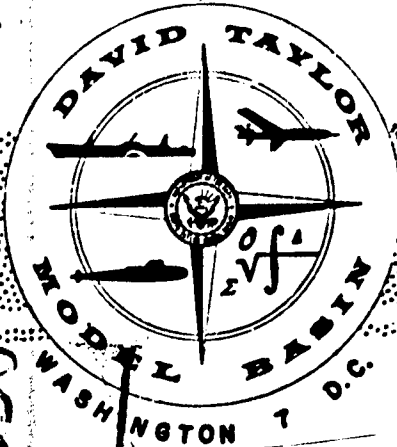
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DEPARTMENT OF THE NAVY

PLASTIC BUCKLING PRESSURE FOR SPHERICAL SHELLS

by

Myron E. Lunchick, Ph.D.

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STRUCTURAL MECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

July 1963

Report 1493

PLASTIC BUCKLING PRESSURE FOR SPHERICAL SHELLS

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**Report 1493
S-F013 03 02**

FOREWORD

The analysis presented in this report was conducted when the author was connected with this activity. Since that time the group now engaged in work on spherical shells have revised the original manuscript, particularly the comparison of results based on the analysis in this report with those based on earlier analyses.

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NOTATION

A_0, A_1, A_{12}	Plasticity coefficients
B_p	Plastic axial rigidity
D_p	Plastic flexural rigidity
E	Young's modulus
E_s	Secant modulus
E_t	Tangent modulus
H	Operator = $\frac{d^2()}{d\theta^2} + \cot \theta \frac{d()}{d\theta} + 2()$
h	Shell thickness
K	Ratio of stress intensity to membrane stress in x-direction = $\sqrt{1 - k + k^2}$
k	Ratio of membrane stress in y-direction to that in x-direction
M_x, M_y	Moments in x- and y-directions, respectively
N_x, N_y	Forces in x- and y-directions, respectively
P	Pressure
P_{cr}	Plastic buckling pressure
P_e	Elastic buckling pressure
P_y	Yield pressure
Q_x	Shear (see Figure 2)
R	Shell radius
S_x, S_y	$E_s \epsilon_x$ and $E_s \epsilon_y$ respectively
u, w	Displacements in x- and z-directions, respectively

x, y, z	Coordinates (see Figure 2)
α	$h^2/12R^2$
γ	Shear strain
ϵ_x, ϵ_y	Strains in x- and y-directions, respectively
ϵ_e	Elastic strain
ϵ_i	Strain intensity
ϵ_p	Plastic strain
ϵ_1', ϵ_2'	Variational membrane strains in x- and y-directions, respectively
η	Plasticity reduction factor
η_B	Bijlaar plasticity reduction factor
η_G	Gerard plasticity reduction factor
η_L	Plasticity reduction factor developed in present paper
θ	Angle (see Figure 2)
μ_e	Elastic Poisson's ratio
μ_s	Secant Poisson's ratio
μ_t	Tangent Poisson's ratio
σ_i	Stress intensity
σ_x, σ_y	Stresses in x- and y-directions, respectively
τ	Shear
ϕ	$\frac{PR(1 - \mu_s^2)}{2E_s h}$

χ_1', χ_2' Variational curvatures in x- and y-directions,
respectively

ψ Angle (see Figure 2)

Primes refer to variational values during the buckling process.

ABSTRACT

A solution for the plastic axisymmetric buckling of thin-walled spheres under hydrostatic pressure is derived. The theory accounts for strain-hardening of material and changes of Poisson's ratio in the plastic range. The plasticity reduction factor is expressed in terms of tangent and secant moduli and new concepts of tangent and secant Poisson's ratios. For typical engineering materials there is little difference between the results obtained from this solution and the earlier ones obtained from the solutions of Bijlaard and Gerard.

INTRODUCTION

Spherical shells have become more prominent in the development of submarines and deep-sea vehicles. Complete spheres are being used for oceanographic research vehicles; the TRIESTE is a notable example. On the other hand, hemispheres are being used to effect closure at bow and stern of advanced submarine designs. Indications are that spherical shells will be used more and more extensively in the future.

A number of investigators have treated the small-deflection analysis of spherical shells subjected to external pressure. Both elastic and inelastic buckling have been studied. Timoshenko^{1*} summarized the classical, linear, small-deflection theory for the elastic buckling

* References are listed on page 17.

pressure of complete spherical shells which was first developed by Zoelly in 1915. An expression for the plastic, small-deflection buckling of spherical shells was first derived by Bijlaard.² Gerard³ obtained an identical expression using deformation theory of plasticity in which the work of Stowell⁴ for flat plates was extended to cylindrical and spherical shells. Both Bijlaard and Gerard in their analyses assumed Poisson's ratio to be equal to a constant, $1/2$. Gerard, however, intuitively modified his expression to include a variable Poisson's ratio.

In this report theory is presented for the plastic, small-deflection buckling of a complete spherical shell of strain-hardening material under external hydrostatic pressure. Whereas Bijlaard² and Gerard³ assumed that Poisson's ratio in the plastic range is a constant, $1/2$, this analysis considers a variable Poisson's ratio. The results of this more rigorous analysis are compared with the existing analysis for spherical shells. A material with an assumed stress-strain relationship is used.

PLASTIC BUCKLING THEORY

VARIATIONS OF FORCES AND MOMENTS

Before the equations of equilibrium of an element within a spherical shell can be established, the variations of the forces and moments resulting from the buckling process must be obtained. The variational forces and moments will be obtained by extending the theory of Reference 5 to spherical shells.

In brief, Reference 5 uses the deformation theory of plasticity generalized for a variable Poisson's ratio. The plasticity theory assumes a monotonically increasing stress-strain curve of the strain-hardening type; see Figure 1. It is assumed that Poisson's ratio is a function of the state of stress and varies from its elastic value $\mu_e = 1/3$ to an upper limit of $\mu_s = 1/2$ for an isotropic, plastically incompressible solid; see Figure 1.

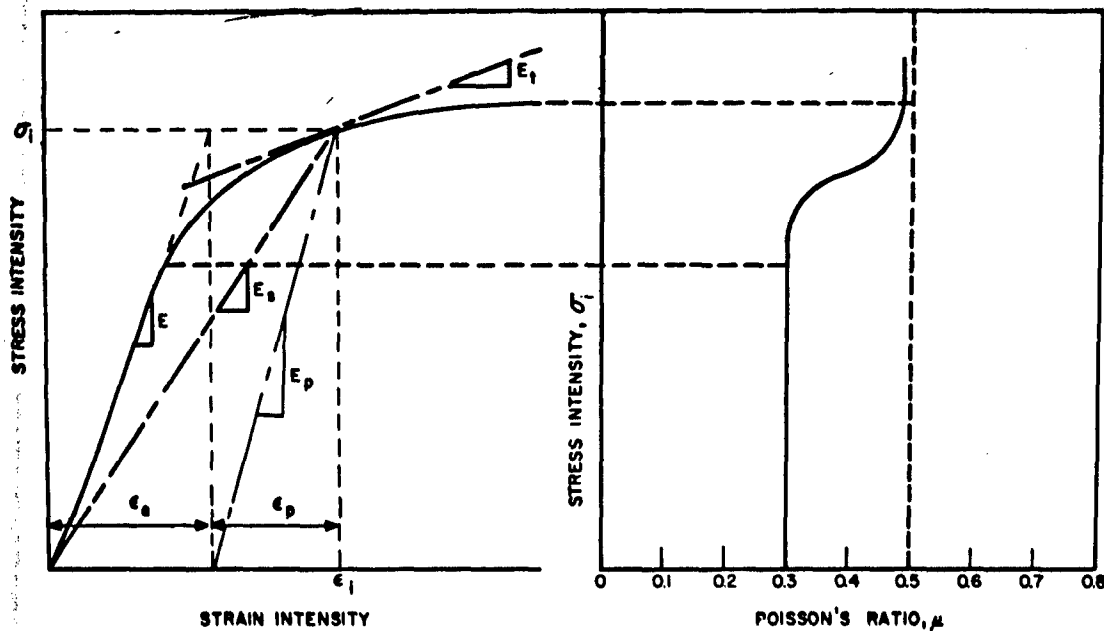


Figure 1 - Variations of Strain and Poisson's Ratio with Stress

The biaxial state of stress and strain in a shell is related to the uniaxial state of stress and strain for a simple compression specimen by expressions for stress and strain intensities. For principal stresses and strains these expressions are as follows:

Stress Intensity:

$$\sigma_i = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y} \quad [1]$$

Strain Intensity:

$$\epsilon_1 = \frac{1}{1 - \mu_s^2} \sqrt{(1 - \mu_s + \mu_s^2)(\epsilon_x^2 + \epsilon_y^2) + (4\mu_s - \mu_s^2 - 1)\epsilon_x \epsilon_y} \quad [2]$$

Stress-Strain Relations:

$$\left. \begin{aligned} \epsilon_x &= \frac{\sigma_x}{E_s} - \mu_s \frac{\sigma_y}{E_s} \\ \epsilon_y &= \frac{\sigma_y}{E_s} - \mu_s \frac{\sigma_x}{E_s} \\ \gamma &= 2(1 + \mu_s) \tau / E_s \end{aligned} \right\} \quad [3]$$

Secant Modulus:

$$E_s = \sigma_i / \epsilon_1 \quad [4]$$

The expression for the plastic Poisson's ratio μ_s is one derived by Gerard and Wildhorn:⁶

$$\mu_s = \frac{1}{2} - \left(\frac{1}{2} - \mu_e\right) \frac{E_s}{E} \quad [5]$$

In Reference 5 these plastic stress-strain relations were used to determine variational forces and moments for the axisymmetric buckling of cylinders. In this presentation, the axisymmetric buckling of spheres will be analyzed. The expression for the variation in strain intensity of

Reference 5 can be readily modified to apply to a sphere. This expression with variations denoted by primes, was found to be:

$$\epsilon'_i = \frac{1}{2(1-\mu_s^2)A_o \sigma_i} \left\{ \left[(2-\mu_s)\sigma_x + (2\mu_s-1)\sigma_y \right] \epsilon'_x + \left[(2\mu_s-1)\sigma_x + (2-\mu_s)\sigma_y \right] \epsilon'_y \right\} \quad [6]$$

where

$$A_o = 1 + \frac{(1 - E_t/E_s)}{4(1-\mu_s^2)K^2} \left\{ \left[(2-\mu_s) - (1-2\mu_s)k \right]^2 - 3(1-\mu_s^2) \right\} \quad [7]$$

For a spherical shell deforming uniformly $\sigma_x = \sigma_y$; therefore $k = \frac{\sigma_y}{\sigma_x} = 1$ and $K^2 = 1$. Also,

$$\epsilon'_x = \epsilon'_1 - z\chi'_1 \quad [8a]$$

$$\epsilon'_y = \epsilon'_2 - z\chi'_2 \quad [8b]$$

Equations [6] and [7] become

$$\epsilon'_1 = \frac{\epsilon'_1 + \epsilon'_2}{2(1-\mu_s)A_o} - \frac{(\chi'_1 + \chi'_2)}{2(1-\mu_s)A_o} z \quad [9]$$

$$A_o = 1 - \frac{(1 - E_t/E_s)(1-2\mu_s)}{2(1-\mu_s)} \quad [10]$$

Let

$$S_x = E_s \epsilon_x \quad [11]$$

Then

$$S'_x = E_s \epsilon'_x - \frac{\epsilon_x}{\epsilon_1} (E_s - E_t) \epsilon'_1 \quad [12]$$

Let $z = z_0$ when $\epsilon_1' = 0$; then, from Equation [9],

$$z_0 = \frac{\epsilon_1' + \epsilon_2'}{\chi_1' + \chi_2'} \quad [13]$$

Substituting Equations [8a], [9], and [13] into Equation [12] gives

$$S_x' = E_s (\epsilon_1' - z \chi_1') + \frac{\epsilon_x (E_s - E_t)}{2(1 - \mu_s) A_o \epsilon_1} (\chi_1' + \chi_2') (z - z_0) \quad [14]$$

Similarly,

$$S_y' = E_s (\epsilon_2' - z \chi_2') + \frac{\epsilon_y (E_s - E_t)}{2(1 - \mu_s) A_o \epsilon_1} (\chi_1' + \chi_2') (z - z_0) \quad [15]$$

$$\text{Since } \sigma_x = \frac{S_x + \mu_s S_y}{1 - \mu_s^2}$$

$$\sigma_x' = \frac{S_x' + \mu_s S_y'}{1 - \mu_s^2} - \frac{[(1 + \mu_s^2) S_y' + 2\mu_s S_x'] \mu_s'}{(1 - \mu_s^2)^2} \quad [16]$$

The variational axial force is

$$N_x' = \int_{-h/2}^{+h/2} \sigma_x' dz \quad [17]$$

Substituting Equation [16] into Equation [17] and integrating gives

$$N_x' = B_p (A_1 \epsilon_1' + \mu_s A_{12} \epsilon_2') \quad [18]$$

where

$$B_p = \frac{E_s h}{1 - \mu_s^2} \quad [19]$$

$$A_1 = 1 - \frac{\left(1 - E_t/E_s\right)(1 + \mu_s)}{4A_o(1 - \mu_s)} \quad [20]$$

$$A_{12} = 1 - \frac{\left(1 - E_t/E_s\right)(1 + \mu_s)}{4\mu_s A_o(1 - \mu_s)} \quad [21]$$

Similarly,

$$N'_y = B_p (A_1 \epsilon'_2 + \mu_s A_{12} \epsilon'_1) \quad [22]$$

The variational axial moment is

$$M'_x = \int_{-h/2}^{+h/2} \sigma'_x z dz \quad [23]$$

Substituting Equation [16] into Equation [23] and integrating,

$$M'_x = -D_p (A_1 \chi'_1 + \mu_s A_{12} \chi'_2) \quad [24]$$

where

$$D_p = \frac{E_s h^3}{1 - \mu_s^2} \quad [25]$$

Similarly,

$$M'_y = -D_p (A_1 \chi'_2 + \mu_s A_{12} \chi'_1) \quad [26]$$

The plasticity coefficients A_1 and A_{12} reduce to expressions obtained by Gerard⁷ when μ_s is set equal to 1/2.

EQUATIONS OF EQUILIBRIUM

The equations of equilibrium for a shell buckling axisymmetrically are given by Timoshenko¹ as:*

$$\left. \begin{aligned} \frac{dN'_x}{d\theta} + (N'_x - N'_y) \cot \theta - Q'_x - \frac{PR}{2} \left(\frac{u}{R} + \frac{dw}{Rd\theta} \right) &= 0 \\ \frac{dQ'_x}{d\theta} + Q'_x \cot \theta + N'_x + N'_y + PR \left(\frac{du}{Rd\theta} + \frac{u}{R} \cot \theta - \frac{2w}{R} \right) \\ - \frac{PR}{2} \left(\frac{du}{Rd\theta} + \frac{d^2w}{Rd\theta^2} \right) - \frac{PR}{2} \cot \theta \left(\frac{u}{R} + \frac{dw}{Rd\theta} \right) &= 0 \\ \frac{dM'_x}{d\theta} + (M'_x - M'_y) \cot \theta - Q'_x R &= 0 \end{aligned} \right\} [27]$$

The notation used is explained in Figure 2. In terms of the displacements,* the variational membrane strains and variational curvatures are:¹

$$\left. \begin{aligned} \epsilon'_1 &= \frac{du}{Rd\theta} - \frac{w}{R} & \epsilon'_2 &= \frac{u}{R} \cot \theta - \frac{w}{R} \\ \chi'_1 &= \frac{d^2w}{R^2 d\theta^2} + \frac{du}{R^2 d\theta} & \chi'_2 &= \left(\frac{u}{R^2} + \frac{dw}{R^2 d\theta} \right) \cot \theta \end{aligned} \right\} [28]$$

Q'_x can be eliminated by combining the first two quantities of Equations [27]

By substituting Equations [18], [22], [24], [26], and [28] into the resulting two equations, there results

* The displacements u and w are variational values but for convenience will not be denoted by primes.

$$\begin{aligned}
& (1 + \alpha) \left[A_1 \frac{d^2 u}{d\theta^2} + A_1 \cot \theta \frac{du}{d\theta} - (\mu_s A_{12} + A_1 \cot^2 \theta) u \right] \\
& - (A_1 + \mu_s A_{12}) \frac{dw}{d\theta} + \alpha \left[A_1 \frac{d^3 w}{d\theta^3} + A_1 \cot \theta \frac{d^2 w}{d\theta^2} - (\mu_s A_{12} + A_1 \cot^2 \theta) \frac{dw}{d\theta} \right] \\
& - \phi \left(u + \frac{dw}{d\theta} \right) = 0 \quad [29]
\end{aligned}$$

and

$$\begin{aligned}
& (A_1 + \mu_s A_{12}) \left(\frac{du}{d\theta} + u \cot \theta - 2w \right) + \alpha \left[-A_1 \frac{d^3 u}{d\theta^3} - 2A_1 \cot \theta \frac{d^2 u}{d\theta^2} \right. \\
& + (A_1 + \mu_s A_{12} + A_1 \cot^2 \theta) \frac{du}{d\theta} - \cot \theta (2A_1 - \mu_s A_{12} + A_1 \cot^2 \theta) u \\
& - A_1 \frac{d^4 w}{d\theta^4} - 2A_1 \cot \theta \frac{d^3 w}{d\theta^3} + (A_1 + \mu_s A_{12} + A_1 \cot^2 \theta) \frac{d^2 w}{d\theta^2} \\
& \left. - \cot \theta (2A_1 - \mu_s A_{12} + A_1 \cot^2 \theta) \frac{dw}{d\theta} \right] \\
& - \phi \left(-u \cot \theta - \frac{du}{d\theta} + 4w + \cot \theta \frac{dw}{d\theta} + \frac{d^2 w}{d\theta^2} \right) = 0 \quad [30]
\end{aligned}$$

where

$$\alpha = \frac{h^2}{12R^2} \quad [31]$$

and

$$\phi = \frac{PR(1 - \mu_s^2)}{2E_s h} \quad [32]$$

Equations [29] and [30] reduce to Equations [e] and [f] on page 492 of Reference 1 when $E_s = E$ and $\mu_s = \mu_e$.

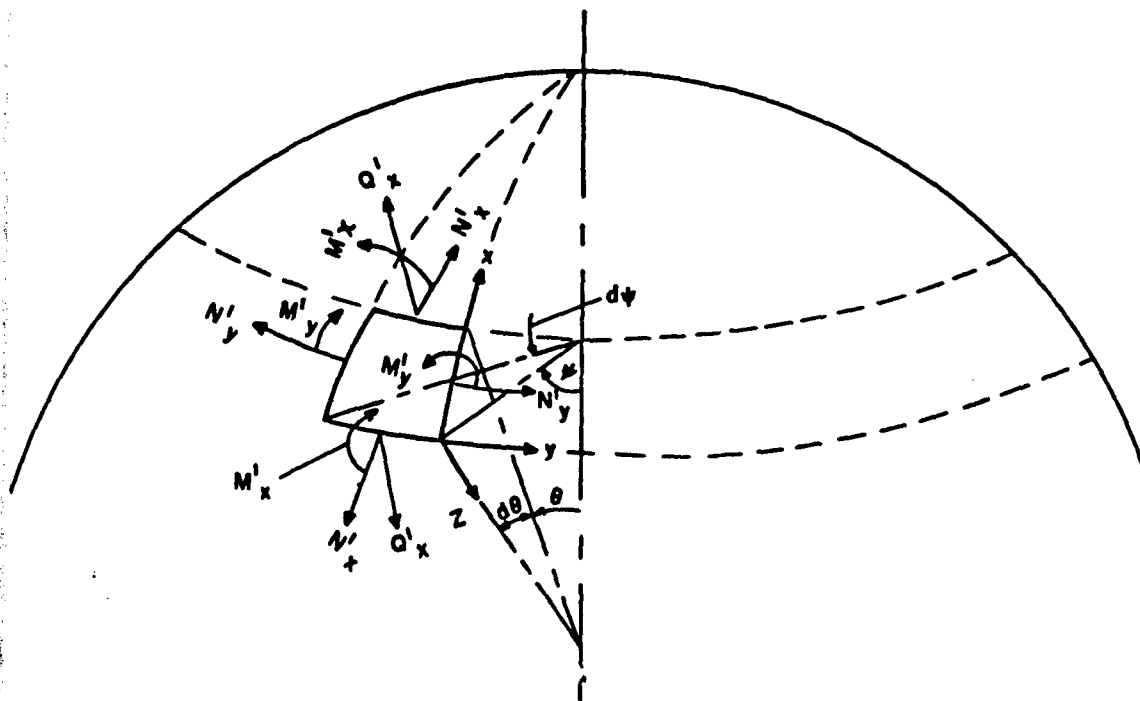


Figure 2 - Forces and Moments on an Element of a Spherical Shell

PLATE BUCKLING EQUATION

The solution for the buckling pressure is practically identical to that described by Timoshenko in Reference 1. Thus, introducing a new variable, the angle ψ (see Figure 2) as follows:

$$u = \frac{d\psi}{d\theta}$$

and using the symbol H for the operation

$$\frac{d^2(\quad)}{d\theta^2} + \cot \theta \frac{d(\quad)}{d\theta} + \gamma' \quad ,$$

there is obtained from Equation [29] (after integrating once) and also from Equation [30] the following:

$$A_1 H(\psi) + \alpha A_1 H(w) - (A_1 + \mu_s A_{12})(\psi + w) - \phi(\psi + w) = 0 \quad [33]$$

$$\begin{aligned} & \alpha A_1 H H(\psi + w) - (A_1 + \mu_s A_{12}) H(\psi) - (3A_1 + \mu_s A_{12}) \alpha H(w + \psi) \\ & + 2(A_1 + \mu_s A_{12})(\psi + w) + \phi[-H(\psi) + H(w) + 2(\psi + w)] = 0 \end{aligned} \quad [34]$$

As in the case of the elastic theory, Equations [33] and [34] have solutions in terms of spherical functions. Rather than repeat steps previously described,¹ only the final buckling equation, resulting from setting the determinant of two homogeneous equations equal to zero, is presented:

$$P_{cr} = \frac{2}{1 - \mu_s^2} \sqrt{\frac{A_1^2 - \mu_s^2 A_{12}^2}{3}} \cdot E_s \left(\frac{h}{R}\right)^2 \quad [35]$$

Equation [35] reduces to the classical, elastic, small-deflection theory $E_s = E$ and $\mu_s = \mu_e$.

The plastic buckling equation is more easily related to the elastic solution if the concepts of secant and tangent Poisson's ratios are introduced. The secant Poisson's ratio is the weighted average of the elastic Poisson's ratio μ_e and the fully plastic value μ_p ; see Figure 1. Thus

$$\mu_s = \frac{\mu_e \epsilon_e + \mu_p \epsilon_p}{\epsilon_1} \quad [36]$$

But $\epsilon_e = \sigma_i / E$ and $\epsilon_p = \sigma_i / E_p$, thus

$$\mu_s = E_s \left(\frac{\mu_e}{E} + \frac{\mu_p}{E_p} \right) \quad [37]$$

Since $\epsilon_p = \epsilon_i - \epsilon_e$, then

$$E_p = \frac{\sigma_i}{\epsilon_i - \epsilon_e} = \frac{\sigma_i}{\frac{\sigma_i}{E_s} - \frac{\sigma_i}{E}} = \frac{1}{\frac{1}{E_s} - \frac{1}{E}}$$

or

$$\frac{1}{E_p} = \frac{1}{E_s} - \frac{1}{E} \quad [38]$$

Substituting Equation [38] into Equation [37],

$$\mu_s = \mu_p - (\mu_p - \mu_e) \frac{E_s}{E} \quad [39]$$

For a fully plastic, isotropic, incompressible material, $\mu_p = 1/2$. Then,

$$\mu_s = \frac{1}{2} - \left(\frac{1}{2} - \mu_e \right) \frac{E_s}{E} \quad [40]$$

Equation [40] is identical to Equation [5] presented by Gerard and Wildhorn⁶ as the plastic Poisson's ratio.

The tangent Poisson's ratio is defined herein as the number which, when multiplied by the variation in strain, gives the variation in strain in the transverse direction. Thus

$$\mu_t \epsilon_1' = (\mu_s \epsilon_1)' = \epsilon_1 \mu_s' + \mu_s \epsilon_1' \quad [41]$$

Therefore,

$$\mu_t = \frac{\epsilon_1 \mu_s' + \mu_s \epsilon_1'}{\epsilon_1'} \quad [42]$$

The variation of Equation [40] is

$$\mu_s' = \left(\frac{1}{2} - \mu_e\right) \frac{\epsilon_1'}{\epsilon_1} (E_s/E - E_t/E) \quad [43]$$

Substituting Equations [40] and [43] into Equation [42] gives the final expression:

$$\mu_t = \frac{1}{2} - \left(\frac{1}{2} - \mu_e\right) \frac{E_t}{E} \quad [44]$$

In terms of the secant and tangent Poisson's ratios, Equations [39] and [44], Equation [35] can be shown to be:

$$P_{cr} = 2 \sqrt{\frac{E_t \cdot E_s}{3(1 - \mu_t)(1 + \mu_s)}} \left(\frac{h}{R}\right)^2 \quad [45]$$

DISCUSSION AND CONCLUSIONS

Equation [45] may be compared with the classical, elastic solution

$$P_e = \frac{2E}{\sqrt{3(1 - \mu^2)}} \left(\frac{h}{R}\right)^2 \quad [46]$$

through the use of the plasticity-reduction factor η defined as

$$\eta = \frac{P_{cr}}{P_e} \quad [47]$$

Substituting Equations [45] and [46] into Equation [47] yields

$$\eta_L = \frac{E_s}{E} \sqrt{\frac{E_t(1 - \mu_e^2)}{E_s(1 - \mu_t)(1 + \mu_s)}} \quad [48]$$

Equation [48] reduces identically to that developed by Bijlaard² and Gerard³ if μ_t and μ_s are set equal to 1/2; thus

$$\eta_{B,G} = \frac{E_s}{E} \sqrt{\frac{E_t(1 - \mu_e^2)}{E_s(0.75)}} \quad [49]$$

Equation [48] also reduces to that proposed by Gerard³ when he intuitively accounted for a variable Poisson's ratio in the plastic range, if μ_t is set equal to μ_s ; thus

$$\eta_G = \frac{E_s}{E} \sqrt{\frac{E_t(1 - \mu_e^2)}{E_s(1 - \mu_s^2)}} \quad [50]$$

Equation [45], therefore, reduces to the following:

1. Zoelly's classical, small deflection, elastic theory when moduli and plastic Poisson's ratios take on elastic values; see Equation [48].
2. Bijlaard's and Gerard's plastic solution when secant and tangent Poisson's ratios are set equal to 1/2; see Equation [49].
3. Gerard's intuitively modified plastic solution when the tangent Poisson's ratio is set equal to the secant Poisson's ratio; see Equation [50].

The collapse pressures as determined by the inelastic theories of Bijlaard and Gerard are compared with Equation [45] of this paper in Figure 3. The abscissa is the ratio of the classical, small-deflection, elastic collapse pressure P_e to the pressure at which the average stress reaches the yield point, P_y , as determined by the 0.2 percent offset method. The ordinate is the ratio of theoretical inelastic collapse pressure P_{cr} , (as determined by Equation [45], Bijlaard and Gerard), to P_y . A typical stress-strain curve for 7075-T6 aluminum bar stock has been assumed in all theoretical calculations. There is little difference between the pressures obtained from the inelastic theories of Bijlaard and Gerard and the more rigorous theory developed herein. The maximum difference between the three theoretical collapse pressures for a spherical shell of 7075-T6 aluminum is about 2 percent; see Figure 3. Similar results would be obtained for other strain-hardening materials.

Theory which fails to consider the effects of imperfections, residual stresses, boundary conditions, and penetrations will not consistently predict the collapse strength of spherical shells normally encountered in engineering practice. However, this theory will, if verified by experiment, serve as a reference to which tests of fabricated, spherical shells may be compared to determine the detrimental effects of initial departures from sphericity, variations in shell thickness, residual stresses, boundary conditions, and penetrations.

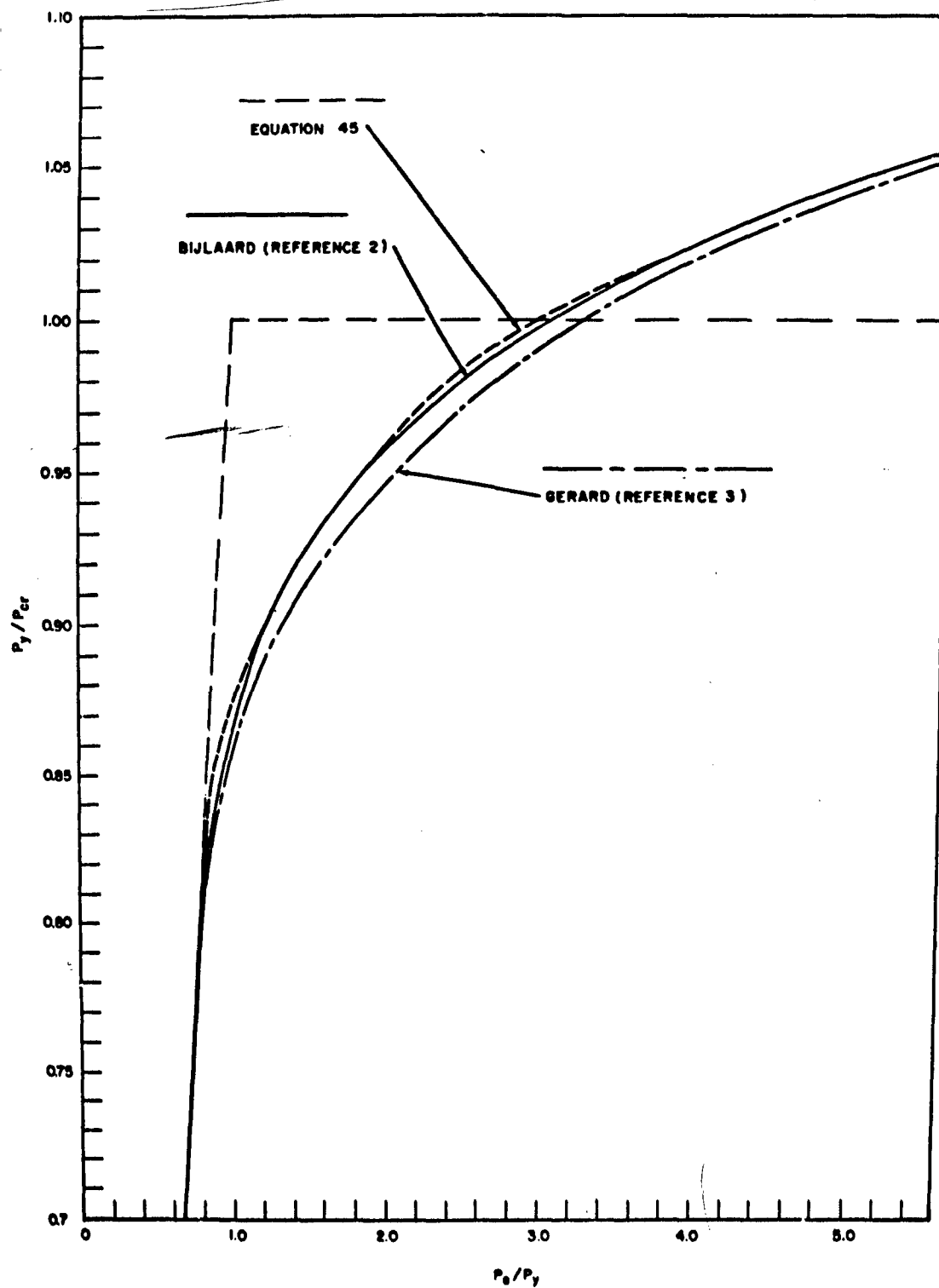


Figure 3 - Comparison of Inelastic Buckling Theories

ACKNOWLEDGMENTS

The author is grateful to Mr. R. D. Short, Jr., of the David Taylor Model Basin for reviewing and checking the analysis. In addition, Mr. Short introduced the concept of tangent Poisson's ratio.

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